EFFECTIVE CROSS-SECTIONS FOR DIELECTRIC WAVEGUIDES IN SUBSTRATE ENVIRONMENTS

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Abstract

The use of an effective cross-section for determining the propagation constants of modes of dielectric waveguides embedded in substrates is illustrated. The resulting expressions are simple, and involve knowledge only of the TM-mode eigenvalues for a similarly shaped hollow waveguide.

Introduction

Because of pecularities in fabrication processes, or because of some desired property of the finished structure, many different shapes of dielectric waveguide have been proposed or used at both optical and millimeter-wave frequencies. The analysis of these various guides has usually been numerical, and can involve substantial amounts of computer time.[1] It would be useful to have simple formulas for such guides that one could use in the design of optical frequency or millimeter-wave devices.

Although results for a few specific structures have been known for awhile, it has recently been demonstrated[2] that propagation constants for the modes of weakly-guiding optical fibers of <u>any</u> cross-section can be accurately predicted (sufficiently far from their cutoff frequencies), by using an "effective" cross-section whose boundary is taken to be a perfect conductor. In this paper, we will extend this result to the case where the cladding region of the waveguide consists of more than one dielectric. This will allow the effective cross-section method to be used for integrated-type waveguides, and considerably extend its usefulness.

The Effective Cross-Section

We have previously shown[2] that a weakly-guiding dielectric waveguide of core refractive index n_1, cladding refractive index n_2, and arbitrary core shape can be analyzed by the method of effective cross-sections. The effective cross-section is defined as the result of pushing the core normally outward at each point of its boundary by a distance δn equal to $k_o^{-1}(n_1^2 - n_2^2)^{-\frac{1}{2}}$, where k_o is the free-space wavenumber $\omega\sqrt{\mu_o \varepsilon_o}$, and ω is the angular frequency of an assumed time dependence of $\exp(i\omega t)$. The process is depicted in Fig. 1. We then find that sufficiently far from cutoff, a mode of the dielectric waveguide propagating as $\exp(-i\beta z)$, where $\beta = k_o n_e$ is the propagation constant and n_e the effective index, obeys the relation

$$\beta^2 \simeq k_1^2 - \tilde{k}_{c,m}^2 \qquad (1)$$

where $k_1 = k_o n_1$ is the wavenumber of the core, and $\tilde{k}_{c,m}$ is the cutoff wavenumber for a TM mode of a hollow metallic waveguide of the <u>effective</u> cross-sectional shape.

The physical interpretation of (1) is clear; δn represents the Goos-Hänchen penetration depth of the fields into the cladding region, in the limit of $\beta \simeq k_1$ (which, in terms of a ray description of the mode, is the limit of grazing ray incidence at the core-cladding boundary). While the derivation of (1) can be done analytically,[2] it seems reasonable to try to extend its validity to more general dielectric waveguides in an empirical fashion, based on this physical interpretation.

The generalization we consider here is to the case when the cladding is inhomogeneous--it consists of more than one dielectric (e.g., when a substrate is present). Let us then define the effective cross-section in this case to be the result of extending the core-cladding boundary normally outward by $\delta n = k_o^{-1}(n_1^2 - n_c^2)^{-\frac{1}{2}}$ at each point, where n_c is now the cladding refractive index (which may be different at different points of the boundary). We will see below that if the waveguide meets the weakly-guiding criterion $(n_1^2 - n_c^2) \ll n_c^2$ for a significant portion of the core boundary, then the result (1) can be used quite accurately for modes of such integrated waveguides as well.

Examples

Consider the rectangular channel guide embedded in a substrate of index n_2 as shown in Fig. 2. The core index again is n_1, and the remaining cladding index is taken to be n_o. With penetration depths and effective cross-section as indicated in Fig. 2, eqn.(1) gives us

$$\frac{n_e^2 - n_2^2}{n_1^2 - n_2^2} \simeq 1 - \left\{ \left(\frac{m\pi}{k_o a\sqrt{n_1^2 - n_2^2} + 2} \right)^2 + \left(\frac{n\pi}{k_o b\sqrt{n_1^2 - n_2^2} + 1 + \Delta} \right)^2 \right\} \qquad (2)$$

where m and n are positive integers and $\Delta = (n_1^2 - n_2^2)^{\frac{1}{2}}/(n_1^2 - n_o^2)^{\frac{1}{2}}$. Equation (2) for the rectangular channel was originally obtained by Marcatili.[3] A comparison of eqn. (2) with a numerical result of Yeh <u>et al.</u>[1] for the fundamental mode is shown in Fig.3. It can be seen that even fairly close to cutoff, the agreement is quite good.

If, in Fig. 2, we regard n_o to be larger than n_2 (i.e., it corresponds to the substrate), then we have a so-called <u>embossed</u> channel guide. Yeh <u>et al.</u>[1] also give results for this case, which are compared with results from the following renormalization of eqn.(2):

$$\frac{n_e^2 - n_o^2}{n_1^2 - n_o^2} \simeq 1 - \left\{ \left(\frac{m\pi}{k_o a\sqrt{n_1^2 - n_o^2} + 2/\Delta} \right)^2 + \left(\frac{n\pi}{k_o b\sqrt{n_1^2 - n_o^2} + 1 + 1/\Delta} \right)^2 \right\} \qquad (3)$$

in Fig.4. Again, we see that agreement is very good, and note as well that the error changes sign as the aspect ratio a/b increases from 1 to 4. This is an

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indication that changes in cross-sectional shape do not severely affect the accuracy of our method.

Another example of an embedded guide is shown in Fig. 5--an embedded right isosceles triangle. The TM modes for a hollow right isosceles triangle waveguide are described by Schelkunoff,[4] and lead us to the expression

$$\frac{n_1^2 - n_e^2}{n_1^2 - n_2^2} \simeq 1 - \frac{\pi^2(m^2 + n^2)}{\left(k_0 a\sqrt{n_1^2 - n_2^2} + 2 + \Delta\sqrt{2}\right)^2} \qquad (4)$$

where m and n are positive integers and Δ is defined as above. Results of eqn. (4) are shown in Fig.6 for the first two mode groups. The author is not aware of any numerical results for this structure which are available for comparison.

Conclusion

The usefulness of the effective cross section method has been demonstrated for a class of dielectric waveguides not possessing a homogeneous cladding region. Many waveguide shapes for which solutions of the corresponding hollow metallic guide are available can be treated using this method, and simple design formulas for unconventional substrate-integrated waveguides can be obtained.

Acknowledgment

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References

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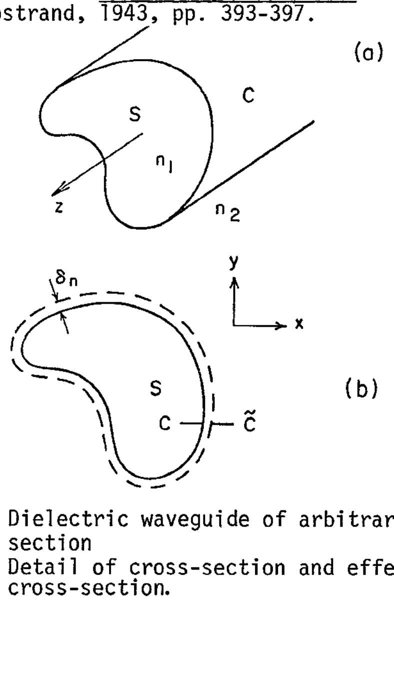

Fig. 1. (a) Dielectric waveguide of arbitrary cross-section
(b) Detail of cross-section and effective cross-section.

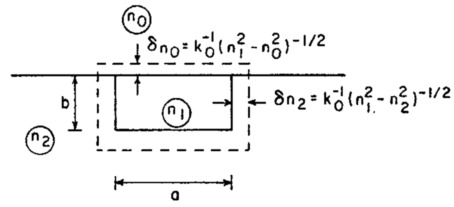

Fig. 2. Rectangular channel in or on a substrate.

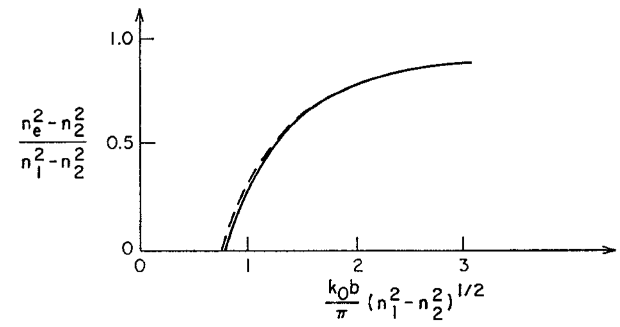

Fig. 3. Fundamental mode dispersion curve for embedded rectangular channel guide: a/b = 2; Δ = 0.274.

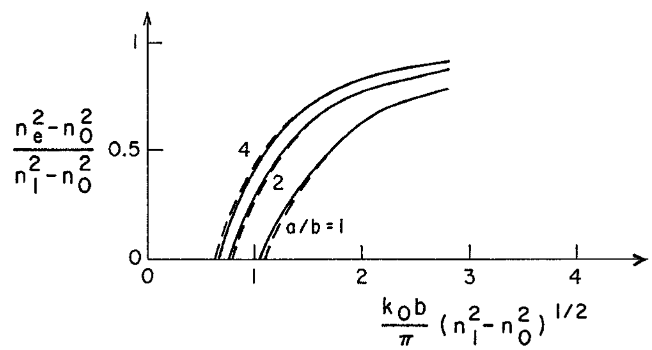

Fig. 4. Fundamental mode dispersion curves for embossed rectangular channel guides: Δ = 2.911.

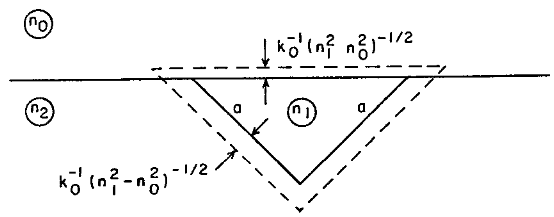

Fig. 5. Embedded right isosceles triangle channel.

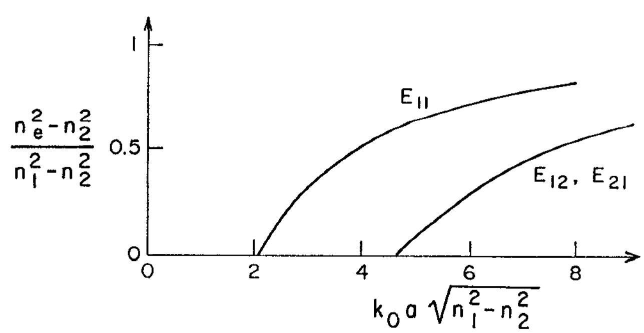

Fig. 6. Fundamental and higher-order mode dispersion curves for embedded right isosceles triangle channel guide: Δ = 0.274.